ABSTRACT

In this paper, a new approach to the design of optimum transmultiplexer filters banks for discrete wavelet multitone multicarrier modulation (DWMT) is presented. DWMT has been proposed for use in high-speed data communications over twisted-pair copper wires. This environment is far from ideal but DWMT is well equipped to deal with channel degradations and noise [1]. Previous techniques have concentrated on optimising the spectral compactness of a prototype filter and rely on a frequency domain analysis of intercarrier interference (ICI) and intersymbol interference (ISI). In this study we account for the time domain overlap between successive pulses by dividing each modulation filter into a series of components in time. In addition we consider a mixed time frequency interpretation of the effects of the channel. An averaged channel model is used in the design of the filters so as we can account for the effects of time dispersion and frequency distortion. We show that the optimisation of the components in the mixed time-frequency domain reduces interference.

1. INTRODUCTION

Figure 1. DWMT system with inputs and corresponding outputs having the same index for simplicity.
biorhythmology being advantageous, it is the possible trade off between system delay and performance. This was illustrated in [3].

2.3 Perfect Reconstruction: If the analysis bank receives signals directly from the synthesis bank, it is desirable that symbols pass through the system without interference i.e. the outputs exactly match the inputs. This is known as the perfect reconstruction (PR) property. It may be best to relax PR constraints with the benefit of more design freedom to combat channel degradations.

2.4 Fast Implementation: Cosine modulated filter banks (CMFB’s) can be implemented efficiently. There is a large price to pay for employing CMFBs, since all filters are derived from a single prototype \( p[n] \), the only design freedom we have is the design of this prototype. This study considers orthogonal, perfect reconstruction, cosine modulated filter banks. From the discussion above it should be clear that these constraints may compromise overall performance for the sake of design ease and implementability. All filters were derived from a prototype \( p[n] \) using the following equation.

\[
h_m[n] = 2 \alpha \cos \left( \frac{\pi}{M} n + 0.5 \right) (n - \frac{gM - 1}{2}) \quad (3)
\]

The prototype is designed using the lattice approach [4], which tends to give highly non-linear optimisation problems. The quadratic constrained formulation described in [5] appears a promising solution.

3. MINIMISING INTERFERENCE

The synthesis filters are of length \( gM \) so each input symbol \( x_m[n] \) contributes to \( gM \) samples outputted from filter \( h_m \). Define these \( gM \) samples as the waveform \( d_m[n] \). Since a new input symbol arrives after \( M \) samples have been outputted, each waveform \( d_m[n] \) will be composed of contributions from symbols \( x_m[k] \) from time frames \( n - g + 1 \leq k \leq n + g - 1 \). Wavesforms from the same filter \( h_m \) overlap in time. All waveforms from time frame \( n \) are added together and passed over the channel. Addition of sequences is a linear operation so waveforms can be considered independent of each other. The received signal is analysed by each analysis filter \( f_m \). If the channel is ideal and the filter bank is PR, output symbols will exactly match input symbols. The \( M^2 \) perfect reconstruction conditions can be described as follows

\[
\sum_{j=0}^{gM-1} \sum_{k=-g}^{g} f_m[N-1-l] d_m'[l] = a_{x_m}[n] (m-j) \quad a_{x_m} = R \quad (4)
\]

Ideally we would like to optimise each waveform with respect to degradations in the channel. However consider the fact that each waveform is a summation of components scaled by past, previous and current symbols, where components are defined as follows.

\[
h_m'[k] = \{0, ..., 0, 0, 1 \} \quad 0 \leq k \leq g \quad (5)
\]

Thus we can define each waveform \( d_m[n] \) as follows

\[
d_m[n] = \sum_{k=0}^{2g} x_m[n+k] h_m[k] e^{-i} \quad (6)
\]

\[d_m[n], h_m[k] \quad \text{are sequences of length } g \times M\]

Since \( x_m[n] \) are uncorrelated random symbols, there are an infinite number of possible waveforms. Hence it is not possible to optimise the waveforms. However if we sub equation (8) into equation (7) we can write the perfect reconstruction conditions in terms of the components

\[
\sum_{j=0}^{gM-1} \sum_{k=-g}^{g} f_m[N-1-l] \sum_{l=0}^{gM-1} x_m[n+k] h_m[k] e^{-i} = a_{x_m}[n] \delta(m-j) \delta(k) \quad (7)
\]

(7) can be rewritten as \((2g-1)M \) orthogonality conditions with \((2g-1)M \) components to optimise. As the components are passed over the channel orthogonality is destroyed. Errors in conditions involving the full filter impulse response \( h_m \) contribute to ICI. Errors in conditions involving all other components contribute to ISI. This is why the traditional technique of optimising each filter leads to excessive ISI. To minimise interference at the receiver we require that distortion of each component due to the channel is limited to a scalar multiplication. The question now arises how do we optimise the components such that this is the case.

4. OPTIMISING COMPONENTS

We need only consider non-zero samples in our analysis so remove all zero samples from components. Each component \( h_m[k] \) is now a sequence of length \( L^2 \) where

\[
L^2 = \frac{kM}{1 \leq k \leq g} \quad (8)
\]

\[
L^2 = \frac{2gM - kM}{g + 1 \leq k \leq 2g - 1}
\]

As a component \( h_m[k] \) is transmitted over a DSL channel, the channel distorts it to give a received component \( r_m[k] \). Since the channel is not flat, the channel impulse response is dispersive. As \( h_m[k] \) is passed over the channel, it will spread out in the time domain. If the discrete model channel is of length \( W \) and \( h_m[k] \) is of length \( L^2 \), the received component \( r_m[k] \) will be of length \( L^2 + L^2 - 1 \). The \( L^2 \) samples \( r_m[k] \) are selected. Ideally we would like \( r_m[k] \) to be equal to \( h_m[k] \), so that the perfect reconstruction conditions are satisfied. However even if this was the case, samples of \( r_m[k] \) outside the window still give an interference term. There are three techniques for reducing the effects of dispersion

1. Increase the length of the components.
2. Use a time domain equaliser to reduce the length of the channel.
3. Design the components so that most of the energy of the received component is concentrated inside the rectangular window.

If the effects of dispersion have been reduced to being negligible

\[
G_m^e(e^{i\theta}) = H_m^e(e^{i\theta}) C(e^{i\theta}) \quad (9)
\]

Decompose equation (9) into amplitude and phase

\[
|G_m^e(e^{i\theta})| = |H_m^e(e^{i\theta})| |C(e^{i\theta})| \quad (10)
\]

\[
\arg(G_m^e(e^{i\theta})) = \arg(H_m^e(e^{i\theta})) + \arg(C(e^{i\theta})) \quad (11)
\]

Ideally we would like to limit distortion to a scalar multiplication so we have zero interference (see section 3). Consider how amplitude and phase distortion, affect this condition.

4.1 Amplitude Distortion

In general the magnitude of the channel frequency response \( |C(e^{i\theta})| \) degrades as the frequency \( \phi \) approaches \( \pi \). If each component \( H_m^e(e^{i\theta}) \) is confined to a single centre frequency \( \omega_m \), equation (14) reduces to

\[
|G_m^e(e^{i\theta})| = H_m^e(e^{i\theta}) \left| C(e^{i\theta}) \right| \quad (12)
\]

and distortion is limited to multiplication by a constant. In this case there is zero amplitude distortion. Generally it is not possible
to confine components to a single frequency due to limited degrees of design freedom. As frequencies are scaled differently distortion is no longer limited to a scalar multiplication and there is interference at the receiver. Rewriting equation (14) as a sum of the ideal term and the error term we get the following
\[ G_n(e^{j\omega}) = H_n(e^{j\omega}) \cdot C(e^{j\omega}) + \left| C(e^{j\omega}) - C(e^{j\omega}) \right| \]

To minimise amplitude distortion, minimise the error term. Assuming frequency degradation we can say that \[ C(e^{j\omega}) - C(e^{j\omega}) \] will increase with \( |\omega_0 - \omega| \). Thus to limit amplitude distortion we need to compact \( |H_n(e^{j\omega})| \) about the centre frequency \( \omega_0 \).

4.2 Phase Distortion. A DSL channel \( C(e^{j\omega}) \) can be approximated as having a linear phase response. This implies all frequencies are approximately delayed by a single time delay \( \tau_0 \). As long as timing recovery is performed correctly, there will be almost negligible phase distortion.

5. NOISE REJECTION

DWMT systems perform excellently in the presence of noise. The longer length of the waveforms gives robustness to impulse noise and the sharper frequency response of the analysis filters greatly mitigate the effects of narrow band noise.

6. CHOICE OF OBJECTIVE FUNCTION

Having considered noise and interference in the DSL environment let us define objective functions to mitigate these effects. From section 2.4, the only design freedom we have is in the selection of a prototype \( P \). If we are considering the frequency response of the filters, we need only consider the prototype. However cosine modulation does not preserve the frequency characteristics of components. If we optimise the frequency response of the components of the prototype, this does not guarantee the optimisation of components of all filters. Hence all filters must be considered when optimising the components. In addition cosine modulation does not preserve any time domain properties so all filters must be considered individually when optimising time domain properties. Filters were optimised using a combination of the two non-linear optimisation techniques Powell and Simplex [6]

6.1 Minimising Stopband Energy: This objective function calculates the ratio of stopband energy to passband energy of the prototype. Note \( P(\theta) \) is normalised to 1 Mathematically we can describe this function as follows
\[ O_1(P) = \int_{-\pi/M}^{\pi/M} \left| \frac{P(e^{j\omega})}{\sum_{k=0}^{M-1} H_n(e^{j\omega})} \right|^2 d\omega \]

6.2 Time Domain Compaction (TDC): This objective function reduces the effects of dispersion in the time domain. Ideally we would like to know the channel impulse response. Instead use an average channel impulse response, derived by considering 5 typical DSL impulse responses at the desired sampling frequency. This approximation is valid because all DSL impulse responses tend to look similar in shape, starting at zero, rising to a peak before decaying back to zero. Each component is passed through the averaged channel and the ratio of the energy inside the desired window to the energy outside is calculated. To find the position of the desired window, the delay \( D \) of the averaged channel was calculated. \( L_2 \) the length of component \( h_{2k} \) is the length of the desired window for \( h_{2k} \). Mathematically we can describe TDC as follows
\[ O_2(p) = \sum_{k=0}^{M-1} \sum_{\omega=0}^{\pi} \left| H_n(e^{j\omega}) \right|^2 \]

6.3 Amplitude Distortion Reduction (ADR). This objective function mitigates the effects of amplitude distortion (see section 4.1). Ideally we would like to know the magnitude of the channel frequency response. As an approximation we use an averaged channel frequency response derived from 5 standard channels. This is a reasonable approximation, as generally DSL channels will degrade with frequency with the greatest variations at lower frequencies. Using the averaged channel we estimate \( |C(e^{j\omega}) - C(e^{j\omega})| \) where \( \omega_0 \) is the centre frequency of the component and \( \omega_0 \) is one element of a finite set of frequencies. Mathematically we can define ADR as follows
\[ O_3(p) = \sum_{k=0}^{M-1} \sum_{\omega=0}^{\pi} |H_n(e^{j\omega}) - C(e^{j\omega})| \]

7. SIMULATIONS AND RESULTS

Two design criteria were considered.

Criterion 1
This is the proposed new design with the components optimised in the mixed time frequency domain. In this case the objective function was chosen as being a weighted sum of \( O_2(P) \) and \( O_3(P) \) objective functions.

Criterion 2
This is the original design with the prototype designed using objective function \( O_2(P) \).

Simulations were performed for 2 ADSL test channels at a sampling frequency of 100 kHz giving a spacing of 2.5kHz. Pulses were modulated at 16 different frequencies in 20 filters to optimise with various degrees of overlap. Table 1 shows the differences in the ratio of stop band energy to passband energy between the two methods.

<table>
<thead>
<tr>
<th>Filter Length</th>
<th>Criterion1</th>
<th>Criterion2</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>4.23 x 10^{-4}</td>
<td>2.59 x 10^{-4}</td>
</tr>
<tr>
<td>80</td>
<td>4.28 x 10^{-4}</td>
<td>3.45 x 10^{-4}</td>
</tr>
<tr>
<td>120</td>
<td>4.81 x 10^{-4}</td>
<td>5.84 x 10^{-4}</td>
</tr>
</tbody>
</table>

Table 1: Prototype: Ratio of Stopband Energy to Passband energy

We employed a 1-tap frequency equaliser at the output of each receive filter. As a measure of design performance we used average signal to interference ratio. After equalisation, output symbols will be composed of signal power due to the correct symbol and signal power due to all other symbols. The ratio of the two is defined as signal to interference ratio. Using a set of 10000
random binary symbols, we observed the signal to interference ratio for each output symbol and calculated the average signal to interference ratio.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Filter Length</th>
<th>Crit.1 SIR (dB)</th>
<th>Crit.2 SIR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t1_601_no13</td>
<td>40</td>
<td>14.7611</td>
<td>14.2008</td>
</tr>
<tr>
<td>t1_601_no13</td>
<td>80</td>
<td>14.7629</td>
<td>14.4775</td>
</tr>
<tr>
<td>t1_601_no13</td>
<td>120</td>
<td>14.7636</td>
<td>14.4631</td>
</tr>
<tr>
<td>csa_no4</td>
<td>40</td>
<td>17.3024</td>
<td>17.0866</td>
</tr>
<tr>
<td>csa_no4</td>
<td>80</td>
<td>17.3220</td>
<td>17.1761</td>
</tr>
<tr>
<td>csa_no4</td>
<td>120</td>
<td>17.3351</td>
<td>17.1449</td>
</tr>
</tbody>
</table>

Table 2: Comparison of observed signal-to-interference ratios for different filter designs

![Log plot of the filter magnitude responses](image1.png)

Figure 2: Modulation filters designed under criterion 1. The magnitude responses of twenty orthogonal filters are shown here, with each filter having 120 taps. Note that the sidelobes of each filter are relatively large in magnitude.

![Log plot of the filter magnitude responses](image2.png)

Figure 3: Modulation filters designed under criterion 2 (minimisation of stopband energy of overall filter). The magnitude responses of twenty orthogonal filters are shown here, with each filter having 120 taps. Note that the sidelobes of each filter are far lower in magnitude than those in Fig. 2.

8. CONCLUSION

We have shown that the optimisation of components gives an average SIR gain of .238dB over the minimisation of stopband energy approach. Optimisation of components was found to be a very challenging non-linear optimisation problem. Future work will concentrate on using more powerful non-linear optimisation techniques. Further improvements would include employing an adaptive training scheme of the filters to replace the averaged channel. The choice of constraints is far from ideal in this study. Biorthogonal filter banks need to be investigated. Also the design of a modulation scheme, which preserves the characteristics of the components of the prototype, would greatly aid future designs.

9. REFERENCES
